Introduction (From MA FW March 2011)

The fundamental purpose of the Model Algebra I course is to formalize and extend the mathematics that students learned in the middle grades. This course is comprised of standards selected from the high school **conceptual categories**, which were written to encompass the scope of content and skills to be addressed throughout grades 9–12 rather than through any single course. Therefore, the complete standard is presented in the model course, with clarifying footnotes as needed to limit the scope of the standard and indicate what is appropriate for study in this particular course. For example, the scope of Model Algebra I is limited to linear, quadratic, and exponential expressions and functions as well as some work with absolute value, step, and functions that are piecewise-defined. Therefore, although a standard may include references to logarithms or trigonometry, those functions are not to be included in coursework for Model Algebra I; they will be addressed later in Model Algebra II. Reminders of this limitation are included as footnotes where appropriate in the Model Algebra I standards.

For the high school Model Algebra I course,[1] instructional time should focus on four critical areas: (1) deepen and extend understanding of linear and exponential relationships; (2) contrast linear and exponential relationships with each other and engage in methods for analyzing, solving, and using quadratic functions; (3) extend the laws of exponents to square and cube roots; and (4) apply linear models to data that exhibit a linear trend.

(1) By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. In Algebra I, students analyze and explain the process of solving an equation and justify the process used in solving a system of equations. Students develop fluency writing, interpreting, and translating among various forms of linear equations and inequalities, and use them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations.

(2) In earlier grades, students define, evaluate, and compare functions, and use them to model relationships between quantities. In Algebra I, students learn function notation and develop the concepts of domain and range. They focus on linear, quadratic, and exponential functions, including sequences, and also explore absolute value, step, and piecewise-defined functions; they interpret functions given graphically, numerically, symbolically, and verbally; translate between representations; and understand the limitations of various representations. Students build on and extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. Students explore systems of equations and inequalities, and they find and interpret their solutions. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

(3) Students extend the laws of exponents to rational exponents involving square and cube roots and apply this new understanding of number; they strengthen their ability to see structure in and create quadratic and exponential expressions. They create and solve equations, inequalities, and systems of equations involving quadratic expressions. Students become facile with algebraic manipulation, including rearranging and collecting terms, and factoring, identifying, and canceling common factors in rational expressions. Students consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. Students expand their experience with functions to include more specialized functions—absolute value, step, and those that are piecewise-defined.

(4) Building upon their prior experiences with data, students explore a more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.

[1] Adapted from the *Common Core State Standards for Mathematics* and *Appendix A: Designing High School Courses based on the Common Core State Standards for Mathematics*

The table below defines sequence of the Algebra I curriculum taught at Winthrop High School. The text below represents an overview of topics covered in this course. For more detailed information the reader should consult the CCSS - MA Frameworks (March 2011) document directly. Relevant content standards are included in the next section for your convenience.

Each quarter's durations should add up to 8 weeks, leaving 1 week of flexibility. The duration serves as a guide to the relative importance of each standard; however, quarterly boundaries are essential for common assessments.

	UNIT	THE STUDENT WILL…	DURATION (WEEKS)
QUARTER 1	***1) Review Pre-Algebra***	a) Perform operations with rational numbers (fractions, decimals & percent) while reviewing order of operations.	3
	2) Solve Equations	a) Model & solve real-life problems involving rational numbers, multi-step equations, proportions, percentages, unit conversions & absolute value.	3
	3) Solve & Graph Inequalities	a) Extend equation solving skills to solve & graph solutions from multi-step, one variable inequality.	2
QUARTER 2	***4) Functions***	a) Perform operations & model real life problems involving function notation & vocabulary from multiple perspectives: graphical, numerical, algebraic & verbal.	2
	5) Linear Functions	a) Model & solve real-life problems involving linear functions. b) Interpret & create standard, y-intercept & point-slope forms of linear equations from multiple perspectives: graphical, numerical, algebraic & verbal.	3
	6) Systems of Linear Equations	a) Model & solve real-life problems involving a system of linear equations from multiple perspectives: graphical, numerical, algebraic & verbal. b) Interpret the solution of a system of linear equations. *May be extended into 3rd quarter at the expense of rational exponents.*	3
QUARTER 3	***7) Exponents & Radicals***	a) Simplify expressions using the rules of exponents. b) Estimate numerical values of numerical radical expressions. c) Extend to rational exponents (optional yet important)	3
	8) Exponential Functions	a) Model & solve real-life problems involving exponential functions. b) Interpret & create exponential functions from multiple perspectives: graphical, numerical, algebraic & verbal.	3
	9) Polynomials	a) Utilize & understand the vocabulary of polynomials b) Perform operations with polynomial expressions: addition, subtraction, multiplication & factoring.	2
QUARTER 4	***10) Quadratic Equations***	a) Model & solve real-life problems involving quadratic equations.	2
	11) Quadratic Functions	a) Interpret & create standard, vertex & factor forms of quadratic equations from multiple perspectives: graphical, numerical, algebraic & verbal. b) Solve quadratic functions using zero product rule, PEMDAS backwards, quadratic formula & graphing.	3
	12) Probability & Statistics	a) Model & solve real-life problems involving experimental & theoretical probability. b) Create, utilize & interpret measures of central tendency in various representations. c) Apply linear modeling to data that exhibits a linear trend.	3

ALGEBRA 1 COMMON CORE CONTENT STANDARDS PROLOG

STANDARDS FOR MATHEMATICAL PRACTICE (SMP)

The Standards for Mathematical Practice should be applied at every opportunity. The *italics* below are my interpretations of the SMP; however, please do read the actual text. The SMP themes are interconnected such that grouping is not only convenient but logical: (1 & 6), (2 & 3), (4 & 5), (7 & 8).

SMP-1 Make sense of problems and persevere in solving them

Allow students the time to try, make and fix mistakes.

SMP-2 Reason abstractly and quantitatively

Let the units of measure lead the way to solving the problem; make the connections between real world and symbols.

SMP-3 Construct viable arguments and critique the reasoning of others

Provide the opportunity for students to listen to their peer's explanations and argument critiques.

SMP-4 Model with mathematics

Provide opportunity for students to use prior knowledge when constructing a mathematical model.

SMP-5 Use appropriate tools strategically

Including but not limited to calculators; "It is a tool, not a crutch!"

SMP-6 Attend to precision

Let this be the over-arching theme: "Does the answer make sence?"

SMP-7 Look for and make use of structure

Allow students the time to recognize and make easier problems by breaking down difficult problems.

SMP-8 Look for and express regularity in repeated reasoning

Provide the opportunity for students to suffer through repeated operations until they discover the short-cut.

NOTATIONS

The + code indicates a standard beyond college and career readiness. These standards should be addressed in accelerated classes

Ro4* is a reference to a collection of standards (detailed at the end of this document) applied to Linear, Exponential and Quadratic functions.

Crossed out ~~content standards~~ are place-holders for future development.

DISCLAIMERS

Remember the transition to CCSS is a work in progress. Our student's success in obtaining the depth and breadth indicated in the following content standards depends on their previous perseverance and academic fortitude. At this time not all content standards may be completely achieved; however, it is our goal to deliver as much content as our students can absorb.

EDITORIAL COMMENTS

According to Dan Meyer the classroom is in need of a make-over. Watch his TED Talk video by searching for "Dan Meyer: Math class needs a makeover".

He recommends providing less information than a typical math textbook problem to gain student "buy-in" and encourage their critical thinking, starting with: "What information do we need to answer this problem?"

He also suggests that math teachers be "less helpful" to encourage students to gain strength in their own problem solving perseverance.

	UNIT	STUDENT OBJECTIVES & COMMON CORE CONTENT STANDARDS	DURATION (WEEKS)
QUARTER 1	***1) Review Pre-Algebra***	a) Perform operations with rational numbers (fractions, decimals & percent) while reviewing order of operations. b) Estimate numerical values of numerical radical and exponential expressions. *N-RN-3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.*	3
	2) Solve Equations	a) Model & solve real-life problems involving rational numbers, multi-step equations, proportions, percentages, unit conversions & absolute value. *A-CED-4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R.* *A-REI-1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.* *N-Q-1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.* *N-Q-2 Define appropriate quantities for the purpose of descriptive modeling.* *N-Q-3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. (MA.3.a Describe the effects of approximate error in measurement and rounding on measurements and on computed values from measurements. Identify significant figures in recorded measures and computed values based on the context given and the precision of the tools used in measure.)*	3
	3) Solve & Graph Inequalities	a) Extend equation solving skills to solve & graph solutions from multi-step, one variable inequality. *A-CED-1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear functions.* *A-REI-3 Solve linear equations and inequalities in one variable, ~~including equations with coefficients represented by letters.~~ (MA.3.a Solve linear equations and inequalities in one variable involving absolute value.)*	2

	Unit	**Student objectives & Common Core Content Standards**	**Duration (Weeks)**
Quarter 2	***4) Functions***	a) Perform operations & model real life problems involving function notation & vocabulary from multiple perspectives: graphical, numerical, algebraic & verbal. *F-BF-3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.* *F-BF-4 Find inverse functions. a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) =2x^3 ~~or $f(x) = (x + 1)/(x – 1)$ for $x \neq 1$.~~* *F-IF-1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.* *F-IF-2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.* *F-IF-3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n + 1) = f(n) + f(n - 1)$ for n greater than or equal to 1.*	2
	5) Linear Functions	a) Model & solve real-life problems involving linear functions. b) Interpret & create standard, y-intercept & point-slope forms of linear equations from multiple perspectives: graphical, numerical, algebraic & verbal. *Ro4* A collection of standards (detailed at the end of this document) that links multiple representations (verbal, analytical, numerical & graphical) of functions and the required skills.* *A-REI-12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), ~~and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half planes.~~*	3
	6) Systems of Linear Equations	a) Model & solve real-life problems involving a system of linear equations from multiple perspectives: graphical, numerical, algebraic & verbal. b) Interpret the solution of a system of linear equations. *A-REI-5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solution.* *A-REI-6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.* *A-REI-11 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions to the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, absolute value, and exponential.*	3

	Unit	Student objectives & Common Core Content Standards	Duration (Weeks)
Quarter 3	***7) Exponents & Radicals***	a) Simplify expressions using the rules of exponents. *N-RN-1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define 5^(1/3) to be the cube root of 5 because we want (5^(1/3))^3=5^(1/3)3 to hold, so (5^(1/3))^3 must equal 5.* *N-RN-2 Rewrite expressions involving radicals ~~and rational exponents~~ using the properties of exponents.*	3
	8) Exponential Functions	a) Model & solve real-life problems involving exponential functions. b) Interpret & create exponential functions from multiple perspectives: graphical, numerical, algebraic & verbal. *Ro4* A collection of standards (detailed at the end of this document) that links multiple representations (verbal, analytical, numerical & graphical) of functions and the required skills.* *A-SSE-1 Interpret expressions that represent a quantity in terms of its context. (a) Interpret parts of an expression, such as terms, factors, and coefficients. (b) Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)^n as the product of P and a factor not depending on P.* *A-SSE-3c Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. (c) Use the properties of exponents to transform expression for exponential functions. For example, the expression 1.15^t can be rewritten as (1.15^(1/12))^12t = 1.012^12t to reveal the approximate monthly interest rate if the annual interest rate is 15%.*	3
	9) Polynomials	a) Utilize & understand the vocabulary of polynomials b) Perform operations with polynomial expressions: addition, subtraction, multiplication & factoring. *A-APR-1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.* *A-SSE-2 Use the structure of an expression to identify ways to rewrite it. For example, see x^4 - y^4 as (x^2)^2 - (y^2)^2, thus recognizing it as the difference of squares that can be factored as (x^2 - y^2)(x^2 + y^2).*	2

	UNIT	STUDENT OBJECTIVES & COMMON CORE CONTENT STANDARDS	DURATION (WEEKS)
QUARTER 4	***10) Quadratic Equations***	a) Model & solve real-life problems involving quadratic equations. *A-REI-4 Solve quadratic equations in one variable. (a) Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x - p)^2 = q that has the same solutions. Derive the quadratic formula from this form. (b) Solve quadratic equations by inspection (e.g., for x^2 = 49), taking square roots, completing the square, the quadratic formula, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex roots. (MA.4.c Demonstrate an understanding of the equivalence of factoring, completing the square, or using the quadratic formula to solve quadratic equations.)*	2
	11) Quadratic Functions	a) Interpret & create standard, vertex & factor forms of quadratic equations from multiple perspectives: graphical, numerical, algebraic & verbal. b) Solve quadratic functions using zero product rule, PEMDAS backwards, quadratic formula & graphing. *Ro4* A collection of standards (detailed at the end of this document) that links multiple representations (verbal, analytical, numerical & graphical) of functions and the required skills.* *A-REI-7 Solve a simple system of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line y = -3x and the circle x^2 + y^2 = 3.* *A-SSE-1 Interpret expressions that represent a quantity in terms of its context. (a) Interpret parts of an expression, such as terms, factors, and coefficients. (b) Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)^n as the product of P and a factor not depending on P.* *A-SSE-3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. (a) Factor a quadratic expression to reveal the zeros of the function it defines. (b) Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.*	3
	12) Probability & Statistics	a) Model & solve real-life problems involving experimental & theoretical probability. b) Create, utilize & interpret measures of central tendency in various representations. c) Apply linear modeling to data that exhibits a linear trend. *S-ID-1 Represent data with plots on the real number line (dot plots, histograms, and box plots).* *S-ID-2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more data sets.* *S-ID-3 Interpret the difference in shape, center, and spread in the context of data sets, accounting for the possible effects of extreme data points (outliers).* *~~S-ID-4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Introduce the use calculators, spreadsheets, and tables to estimate the areas under the normal curve~~* *S-ID-5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.* *~~S-ID-6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. (a) Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. (b) Informally assess the fit of a function by plotting and analyzing residuals. (c) Fit a linear function for a scatter plot that suggests a linear association.~~* *~~S-ID-7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.~~* *~~S-ID-8 Compute (using technology) and interpret the correlation coefficient of a linear fit.~~* *~~S-ID-9 Distinguish between correlation and causation.~~*	3

	UNIT	STUDENT OBJECTIVES & COMMON CORE CONTENT STANDARDS	DURATION (WEEKS)
*	*Rule of 4*	a) The following standards are applicable to the 3 major functions (linear, exponential and quadratic) of Algebra 1.The standards are grouped together here to minimize redundancy of separate listings in linear, exponential and quadratic units. *A-APR-1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.* *A-CED-1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from simple rational and exponential functions.* *A-CED-2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.* *A-REI-10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a cure (which could be a line).* *F-IF-4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* *F-IF-5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* *F-IF-6 Calculate and interpret the average rate of change of a function (represented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.* *F-IF-7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. (a) Graph linear and quadratic functions and show intercepts, maxima, and minima. (b) Graph piecewise-defined functions, including step functions and absolute value functions. (e) Graph exponential showing intercepts.* *F-IF-8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. (a) Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of context. (MA.8.c Translate among different representations of functions and relations: graphs, equations, point sets, and tables.)* *F-IF-9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one function and an algebraic expression for another, say which has a larger intercept.* *F-IF-MA-10 Given algebraic, numeric and/or graphical representations of functions, recognize the function as polynomial, rational, or exponential.* *F-LE-1 Distinguish between situations that can be modeled with linear functions and with exponential functions. (a) Prove that linear functions grown by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. (b) Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. (c) Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.* *F-LE-2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).* *F-LE-3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.* *F-LE-5 Interpret the parameters in a linear function in terms of a context.*	Not all content standards apply to all instances of "Rule of 4"